

Oef 18 p A.31-32

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & a \\ 0 & 2 & 2 & 3 \\ 1 & 1 & a & -1 \end{pmatrix}$$

$$(a) \begin{matrix} R_2 + R_1 \\ \sim \\ R_4 - R_1 \end{matrix} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & \textcircled{2} & -1 & a \\ 0 & 2 & 2 & 3 \\ 0 & 2 & a & -1 \end{pmatrix}$$

$$\begin{matrix} R_3 - R_2 \\ \sim \\ R_4 - R_2 \end{matrix} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 2 & -1 & a \\ 0 & 0 & \textcircled{3} & 3-a \\ 0 & 0 & a+1 & -1-a \end{pmatrix}$$

$$\begin{matrix} R_4 - \frac{(a+1)}{3} R_3 \\ \sim \end{matrix} \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 2 & -1 & a \\ 0 & 0 & 3 & 3-a \\ 0 & 0 & 0 & -1-a - \frac{a+1}{3}(3-a) \end{pmatrix}$$

$$(a+1) \left(\frac{-1 \cdot 3 - (3-a)}{3} \right) = (a+1)(a-6)$$

$$\sim \begin{pmatrix} 1 & -1 & 0 & 3 \\ 0 & 2 & -1 & a \\ 0 & 0 & 3 & 3-a \\ 0 & 0 & 0 & \frac{1}{3}(a+1)(a-6) \end{pmatrix}$$

A is sing $\Leftrightarrow a = -1$ of $a = 6$

$$\begin{aligned}
 (b) \det A &= \det(\text{gered. vorm}) \\
 &= 1 \cdot 2 \cdot 3 \cdot \frac{1}{3} (a-1)(a-6) \\
 &= 2(a+1)(a-6)
 \end{aligned}$$

$$\begin{aligned}
 A \text{ is sing} &\Leftrightarrow \det A = 0 \\
 &\Leftrightarrow a = -1 \text{ of } a = 6
 \end{aligned}$$

$$(c) \begin{vmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & a \\ 0 & 2 & 2 & 3 \\ 1 & 1 & a & -1 \end{vmatrix}$$

$$\begin{aligned}
 &\stackrel{\text{outw}}{=} \begin{vmatrix} 1 & (-1)^{1+1} & 3 & -1 & a \\ & & 2 & 2 & 3 \\ & & 1 & a & -1 \end{vmatrix} - 1(-1)^{1+2} \begin{vmatrix} -1 & -1 & a \\ 0 & 2 & 3 \\ 1 & a & -1 \end{vmatrix} \\
 &= 3(-1)^{1-1} \begin{vmatrix} 2 & 3 \\ a & -1 \end{vmatrix} + 2(-1)^{2+1} \begin{vmatrix} -1 & a \\ a & -1 \end{vmatrix} + 1(-1)^{3+1} \begin{vmatrix} -1 & a \\ 2 & 3 \end{vmatrix} + (-1)(-1)^{4+1} \begin{vmatrix} 2 & 3 \\ a & -1 \end{vmatrix} + 1(-1)^{3+1} \begin{vmatrix} -1 & a \\ 2 & 3 \end{vmatrix} \\
 &= 2(-2-3a) - 2(1-a^2) + 2(-3-2a) \\
 &= 2a^2 - 10a - 12 \\
 &= 2(a^2 - 5a - 6) \\
 &= 2(a+1)(a-6)
 \end{aligned}$$