Oef 16 p A 31

$$A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$$

?a is A singulier

$\Updownarrow$

det $A = 0$

$\Updownarrow$

er is minstens 1 0-rij i/d gereduceerde vorm

Meth1:

$$|A| = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} \overset{K_1+K_2+K_3}{=} \begin{vmatrix} a+2 & 1 & 1 \\ a+2 & a & 1 \\ a+2 & 1 & a \end{vmatrix} = (a+2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix}$$

$$\overset{\substack{R_2-R_1 \\ R_3-R_1}}{=} (a+2) \begin{vmatrix} 1 & 1 & 1 \\ 0 & a-1 & 0 \\ 0 & 0 & a-1 \end{vmatrix} = (a+2)(a-1)^2$$

A sing $\Leftrightarrow$ det $A = 0$

$\Leftrightarrow a = -2$ of $a = 1$

Meth2:

$$|A| = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{vmatrix} \overset{R_1 \leftrightarrow R_2}{=} - \begin{vmatrix} 1 & a & 1 \\ a & 1 & 1 \\ 1 & 1 & a \end{vmatrix}$$

$$\overset{\substack{R_2-aR_1 \\ R_3-R_1}}{=} - \begin{vmatrix} 1 & a & 1 \\ 0 & 1-a^2 & 1-a \\ 0 & 1-a & a-1 \end{vmatrix} \overset{K_1}{=} - \begin{vmatrix} 1-a^2 & 1-a \\ 1-a & a-1 \end{vmatrix}$$

$$= -(1-a)^2 \begin{vmatrix} 1+a & 1 \\ 1 & -1 \end{vmatrix}$$

$$= -(1-a)^2(-1-a-1)$$

$$= -(1-a)^2(-a-2)$$

$$= (a+2)(a-1)^2$$

<u>Meth 3</u> : door reduceren v. A

$$\begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix} \overset{R_1 \leftrightarrow R_2}{\sim} \begin{pmatrix} 1 & a & 1 \\ a & 1 & 1 \\ 1 & 1 & a \end{pmatrix}$$

$$\underset{R_3-R_1}{\overset{R_2-aR_1}{\sim}} \begin{pmatrix} 1 & a & 1 \\ 0 & 1-a^2 & 1-a \\ 0 & 1-a & a-1 \end{pmatrix} \overset{R_2 \leftrightarrow R_3}{\sim} \begin{pmatrix} 1 & a & 1 \\ 0 & 1-a & a-1 \\ 0 & 1-a^2 & 1-a \end{pmatrix}$$

* $1-a=0 \iff a=1$

$$\sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow$$ A singulier

* $1-a \neq 0$

$$\underset{\frac{1}{1-a}R_3}{\overset{\frac{1}{1-a}R_2}{\sim}} \begin{pmatrix} 1 & a & 1 \\ 0 & 1 & -1 \\ 0 & 1+a & 1 \end{pmatrix}$$

$$\overset{R_3-(a+1)R_2}{\sim} \begin{pmatrix} 1 & a & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1+(a+1) \end{pmatrix} \sim \begin{pmatrix} 1 & a & 1 \\ 0 & 1 & -1 \\ 0 & 0 & a+2 \end{pmatrix}$$

A is sing als

$a+2=0 \iff a=-2$