

Oef 1 (54)

$$\int \frac{dx}{x^3 + 5x^2 + 4x}$$

Step 1 gr T \geq gr N? Nee
 0 3

Step 2

1) Ontbinding v/d N:

$$\begin{aligned} x^3 + 5x^2 + 4x &= x(x^2 + 5x + 4) \\ &= x(x+1)(x+4) \end{aligned}$$

2) Splitting in PB

$$\begin{aligned} \frac{1}{x^3 + 5x^2 + 4x} &= \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+4} \\ &= \frac{A(x+1)(x+4) + Bx(x+4) + Cx(x+1)}{x(x+1)(x+4)} \\ &= \frac{(A+B+C)x^2 + (5A+4B+C)x + 4A}{x(x+1)(x+4)} \end{aligned}$$

$$\begin{cases} A+B+C=0 \\ 5A+4B+C=0 \\ 4A=1 \end{cases} \Leftrightarrow \begin{cases} A = \frac{1}{4} \\ B+C = -\frac{1}{4} \\ 4B+C = -\frac{5}{4} \end{cases} \begin{array}{l} N \\ -1 \\ 1 \end{array} \begin{array}{l} +4 \\ -1 \\ -1 \end{array}$$

$$\Leftrightarrow \begin{cases} A = \frac{1}{4} \\ 3B = -1 \Rightarrow B = -\frac{1}{3} \\ 3C = \frac{1}{4} \Rightarrow C = \frac{1}{12} \end{cases}$$

$$\begin{aligned}\int \frac{dx}{x^3+5x^2+4x} &= \int \left(\frac{\frac{1}{4}}{x} + \frac{-\frac{1}{3}}{x+1} + \frac{\frac{1}{12}}{x+2} \right) dx \\ &= \frac{1}{4} \int \frac{dx}{x} - \frac{1}{3} \int \frac{dx}{x+1} + \frac{1}{12} \int \frac{dx}{x+2} \\ &= \frac{1}{4} \ln|x| - \frac{1}{3} \ln|x+1| + \frac{1}{12} \ln|x+2| + C \\ &= \frac{1}{12} \left(\ln|x|^3 - 4 \ln|x+1| + \ln|x+2| \right) + C \\ &= \frac{1}{12} \ln \left(\frac{|x|^3 |x+2|}{|x+1|^4} \right) + C\end{aligned}$$