Oef 7

$f(x) = x \ln(x+3)$, $x+3>0$

(a) $\int \underset{\underset{g'(x)}{\parallel}}{x} \underbrace{\ln(x+3)}_{f(x)} dx$

$g'(x) = x \Rightarrow g(x) = \frac{x^2}{2}$

$= \frac{1}{2} \int \ln(x+3)\, d\, x^2$

$\overset{PI}{=} \frac{1}{2} \left[x^2 \ln(x+3) - \int x^2 \cdot \frac{1}{x+3} dx \right]$

$= \frac{1}{2} \left[x^2 \ln(x+3) - \int \frac{x^2}{x+3} dx \right]$

$$\begin{array}{r|l} x^2 & x+3 \\ \hline -\ (x^2+3x) & x-3 \\ -3x & \\ -\ (-3x-9) & \\ 9 & \end{array}$$

$= \frac{1}{2} \left[x^2 \ln(x+3) - \int (x-3) dx - 9 \int \frac{dx}{x+3} \right]$

$= \frac{1}{2} \left[x^2 \ln(x+3) - \frac{(x-3)^2}{2} - 9 \ln(x+3) \right]$

$= \frac{(x^2-9)}{2} \ln(x+3) - \frac{1}{4}(x-3)^2$

(b) $I = \int_{-3}^{-3+e^3} f(x)\,dx$

(i) $x = -3+e^3$ $\lim\limits_{\substack{x \to -3+e^3 \\ <}} x \ln(x+3)$

$= (-3+e^3) \ln e^3 = \underset{\in \mathbb{R}}{3(-3+e^3)}$

$\Rightarrow$ eigenlijk mbt bovengrens

$x = -3$ $\lim\limits_{\substack{x \to -3 \\ >}} x \ln(x+3) = -3(-\infty) = +\infty$

$\Rightarrow$ onegenlijk mbt ondergrens

$I = \lim\limits_{\substack{t \to -3 \\ >}} \int_t^{-3+e^2} f(x)\,dx$

$I = \lim\limits_{\substack{t \to -3 \\ >}} \left[\frac{1}{2}(x^2-9)\ln(x+3) - \frac{1}{4}(x-3)^2 \right]_t^{-3+e^3}$

$\overset{\lim_{t \to -3}}{=} \left[\frac{1}{2}(-3+e^3-3)(e^3)\ln(e^3) - \frac{1}{4}(-6+e^3)^2 - \left(\frac{1}{2}(t^2-9)\ln(t+3) - \frac{1}{4}(t-3)^2 \right) \right]$

$= \frac{3}{2}(-6+e^3)e^3 - \frac{1}{4}(36-12e^3+e^6) - \frac{1}{2} \lim\limits_{\substack{t \to -3 \\ >}} (t^2-9)\ln(t+3) + \frac{1}{4}36$

$= \left(\frac{3}{2} - \frac{1}{4}\right)e^6 - 6e^3 - \frac{1}{2}(-6) \lim\limits_{\substack{t \to -3 \\ >}} \frac{\ln(t+3)}{\frac{1}{t+3}}$

$$= \frac{5}{4}e^6 - 6e^3 + 3 \lim_{\substack{t \to -3 \\ >}} \frac{\ln(t+3)}{\frac{1}{t+3}}$$

$$\overset{H}{=} \frac{5}{4}e^6 - 6e^3 + 3 \lim_{\substack{t \to -3 \\ >}} \frac{\frac{1}{t+3}}{-\frac{1}{(t+3)^2}}$$

$$= \frac{5}{4}e^6 - 6e^3 \cancel{+} - 3 \underbrace{\lim_{\substack{t \to -3 \\ >}} (t+3)}_{=0}$$

$$= \frac{5}{4}e^6 - 6e^3 \in \mathbb{R}$$

$\Rightarrow$ convergent