Oef 6

$$f : x \longrightarrow \frac{x^3+x^2}{x^4-1}$$

$$f(x) = \frac{x^3+x^2}{x^4-1}$$

(a) $$\int \frac{x^3+x^2}{x^4-1}\, dx$$

$$N: x^4-1 = (x^2-1)(x^2+1) = (x-1)(x+1)(x^2+1)$$

$$\int \frac{x^2\cancel{(x+1)}}{(x-1)\cancel{(x+1)}(x^2+1)}\, dx$$

PB: $$\frac{x^2}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$= \frac{A(x^2+1)+(Bx+C)(x-1)}{(x-1)(x^2+1)}$$

$$\begin{cases} A+B=1 \\ -B+C=0 \\ A-C=0 \end{cases} \Leftrightarrow \begin{cases} C=A \\ B=A \\ 2A=1 \end{cases} \Leftrightarrow \begin{cases} A=\frac{1}{2} \\ B=\frac{1}{2} \\ C=\frac{1}{2} \end{cases}$$

$$= \frac{1}{2}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{x+1}{x^2+1}\, dx$$

$$= \frac{1}{2}\ln|x-1| + \frac{1}{2}\int \frac{\frac{1}{2}(2x)+1}{x^2+1}\, dx$$

$$= \frac{1}{2}\ln|x-1| + \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\,\mathrm{bgtg}\, x$$

$$= \frac{1}{4}\ln\left[(x-1)^2(x^2+1)\right] + \frac{1}{2}\,\mathrm{bgtg}\, x$$

(b) $I = \int_{-1}^{1} f(x)\,dx$

(i) $x = 1$ $\quad \lim\limits_{\substack{x \to 1 \\ <}} \frac{x^3 + x^2}{x^4 - 1} = \frac{2}{0} = \infty$

$\Rightarrow$ oneigenl m.b.t. bovengrens

$x = -1$ $\quad \lim\limits_{\substack{x \to -1 \\ >}} \frac{x^3 + x^2}{x^4 - 1} = \lim\limits_{\substack{x \to -1 \\ >}} \frac{x^2}{(x-1)(x^2+1)}$

$= \frac{1}{-2 \cdot 2} = -\frac{1}{4} \in \mathbb{R}$

$\Rightarrow$ niet oneigenl m.b.t. ondergrens

$I = \lim\limits_{\substack{t \to 1 \\ <}} \int_{-1}^{t} f(x)\,dx$

(ii) $I = \lim\limits_{\substack{t \to 1 \\ <}} \left[\frac{1}{2} \ln|x-1| + \frac{1}{4} \ln|x^2+1| + \frac{1}{2} \operatorname{bgtg} x \right]_{-1}^{t}$

$= \lim\limits_{\substack{t \to 1 \\ <}} \left(\frac{1}{2} \ln|t-1| + \frac{1}{4} \ln(t^2+1) + \frac{1}{2} \operatorname{bgtg} t - \left(\frac{1}{2} \ln 2 + \frac{1}{4} \ln 2 + \frac{1}{2} \operatorname{bgtg}(-1) \right) \right)$

$= \frac{1}{2}(-\infty) + \cancel{\frac{1}{4} \ln 2} + \frac{1}{2} \frac{\pi}{4} - \frac{1}{2} \ln 2 - \cancel{\frac{1}{4} \ln 2} + \frac{1}{2} \frac{\pi}{4}$

$= -\infty \Rightarrow$ divergent !