

$\sqrt{a^2-x^2}$  in integrandum.

$$\begin{aligned} u = \sqrt{a^2-x^2} &\Rightarrow u^2 = a^2-x^2 \\ &\Rightarrow x = \sqrt{a^2-u^2} \end{aligned}$$

$$x = a \sin t$$

$$a^2 - x^2 = a^2 - a^2 \sin^2 t = a^2 (1 - \sin^2 t) = a^2 \cos^2 t$$

$$\sqrt{a^2-x^2} = a \cos t$$

Def 1 (31)

$$\int \sqrt{a^2-x^2} dx = \int a \cos t \cdot a \cos t dt$$

$$x = a \sin t \Rightarrow dx = a \cos t dt$$

$$\sqrt{a^2-x^2} = a \cos t \Rightarrow \sin t = \frac{x}{a} \Rightarrow t = \arcsin\left(\frac{x}{a}\right)$$

$$= a^2 \int \cos^2 t dt$$

$$= a^2 \int \frac{\cos(2t) + 1}{2} dt$$

$$= \frac{a^2}{2} \int \cos(2t) dt + \frac{a^2}{2} t$$

$$= \frac{a^2}{4} \sin 2t + \frac{a^2}{2} t + C$$

$$= \frac{a^2}{2} \sin t \cos t + \frac{a^2}{2} t + C$$

$$= \frac{1}{2} x \sqrt{a^2-x^2} + \frac{a^2}{2} \arcsin\left(\frac{x}{a}\right) + C$$