Oef 4 (f)

$$\int_1^{+\infty} \frac{1}{z(z^2+1)^2}\,dz$$

1) Oneigenl?

$+\infty \to$ oneigenl mbt bovengrens

$z=1 \quad \lim\limits_{\substack{z \to 1 \\ >}} \frac{1}{z(z^2+1)^2} = \frac{1}{4} \Rightarrow$ Neen

$$I = \lim_{t \to +\infty} \int_1^t \frac{1}{z(z^2+1)^2}\,dz$$

Onbep. int. : $\displaystyle\int \frac{dz}{z(z^2+1)^2}$

PB: $\displaystyle\frac{1}{z(z^2+1)^2} = \frac{A}{z} + \frac{Bz+C}{\dots} + \frac{Dz+E}{(z^2+1)^2}$

$$= \frac{A(z^4+2z^2+1) + (Bz+C)(z^3+z) \circledast}{z(z^2+1)^2}$$

$\circledast \ (Dz+E)z$

$$\begin{cases} A+B=0 \\ C=0 \\ 2A+B+D=0 \\ C+E=0 \\ A=1 \end{cases} \Leftrightarrow \begin{cases} A=1 \\ B=-1 \\ C=0 \\ D=-2A-B=-1 \\ E=0 \end{cases}$$

$$= \int \frac{1}{z}\,dz \cancel{+} - \int \frac{z\,dz}{z^2+1} - \int \frac{z}{(z^2+1)^2}\,dz$$

$$= \ln|z| - \frac{1}{2}\int \frac{du}{u} - \frac{1}{2}\int \frac{du}{u^2}$$

$z^2+1=u$

$2z\,dz=du$

$$= \ln|z| - \frac{1}{2}\ln|z^2+1| + \frac{1}{2}\,\frac{1}{z^2+1}$$

$$= \frac{1}{2}\ln\left|\frac{z^2}{z^2+1}\right| + \frac{1}{2}\,\frac{1}{z^2+1}$$

$$I = \lim_{t\to+\infty}\left[\frac{1}{2}\ln\left(\frac{t^2}{t^2+1}\right) + \frac{1}{2}\,\frac{1}{t^2+1} - \left(\frac{1}{2}\ln\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\overset{0}{\cancel{\ln 1}} + \frac{1}{2}\cdot 0 + \frac{1}{2}\ln 2 - \frac{1}{4}$$

$$= \frac{1}{2}\ln 2 - 4 \in \mathbb{R} \Rightarrow \text{Convergent!}$$