③ Partiele integratie

$$\cancel{\int f(x)\,g(x)\,dx = \int f(x)dx \cdot \int g(x)\,dx}$$

$$\int f(x)\cdot g'(x)\,dx = f(x)\,g(x) - \int g(x)\,f'(x)\,dx$$

$$\int f(x)\,d\,g(x) = f(x)\,g(x) - \int g(x)\,d\,f(x)$$

$$\int f\,dg = fg - \int g\,df$$

Wanneer?

→ exp × macht van x

→ sin, cos × " " "

→ ln(x) × " " "

Oef 1 (16)

$$\int x^n e^x\,dx\,,\quad n = 1, 2, 3, \dots$$

$n=1$: $\int \underset{f(x)}{x^1}\,\underset{g'(x)}{e^x}\,dx \Rightarrow g(x) = e^x$

$$= \int x\,d\,e^x$$

$$\overset{PI}{=} x e^x - \int 1\cdot e^x\,dx$$

$$= x e^x - e^x + C_1 = (x-1)e^x + C_1$$

$n=2$: $\int \underset{f(x)}{x^2}\,\underset{g'(x)}{e^x}\,dx = \int x^2\,d(e^x)$

$$\overset{PI}{=} x^2 e^x - \int e^x\,dx^2$$

$$= x^2 e^x - 2\int x^1 e^x\,dx$$

$$= x^2 e^x - 2(x-1)e^x + C_1$$

Oef 1. (15)

$$\int e^{ax} \cos bx \, dx$$

$e^{ax} = g'(x)$, $\cos bx = f(x)$

$$g'(x) = e^{ax} \Rightarrow g(x) = \frac{e^{ax}}{a}$$

$$= \frac{1}{a}\int \cos bx \, d\left(e^{ax}\right)$$

$$\overset{PI}{=} \frac{1}{a}\left[e^{ax}\cos bx - \int e^{ax}(-\sin(bx) \cdot b)\,dx\right]$$

$$= \frac{1}{a}\left[e^{ax}\cos bx + b\int e^{ax}\sin bx \, dx\right]$$

$e^{ax} = g'(x)$, $\sin bx = f(x)$

$$= \frac{1}{a}\left[e^{ax}\cos bx + \frac{b}{a}\int \sin bx \, d\,e^{ax}\right]$$

$$\overset{PI}{=} \frac{1}{a}\left[e^{ax}\cos bx + \frac{b}{a}\left(e^{ax}\sin bx - \int e^{ax}\cos(bx) \cdot b \, dx\right)\right]$$

$$I = \frac{1}{a}\left[e^{ax}\cos bx + \frac{b}{a}e^{ax}\sin bx - \frac{b^2}{a}\underbrace{\int e^{ax}\cos bx \, dx}_{I}\right]$$

$$\left(1 + \frac{b^2}{a^2}\right)I = \frac{1}{a}e^{ax}\cos bx + \frac{b}{a^2}e^{ax}\sin bx$$

$$I = \frac{a^2}{a^2+b^2}\left(\frac{1}{a}e^{ax}\cos bx + \frac{b}{a^2}e^{ax}\sin bx\right)$$

$$= \frac{a}{a^2+b^2}e^{ax}\cos bx + \frac{b}{a^2+b^2}e^{ax}\sin bx + C$$