# Oef 8(j) p A.15

$$f(x) = \sqrt{x}\ln x$$

(i) def(f)

VW$^{\underline{t}}$: 1) $x \geq 0$

2) def(ln) $= ]0, +\infty[ \Rightarrow x > 0$

$\Rightarrow$ def(f) $= ]0, +\infty[$

(ii) geen symmetrie want

def(f) is niet symm. tov 0.

(iii) Tekenverloop

| $x$ | | 0 | | 1 | |
|---|---|---|---|---|---|
| $\sqrt{x}$ | // | 0 | + | + | + |
| $\ln x$ | // | \| | − | 0 | + |
| $f(x)$ | // | \| | − | 0 | + |

(iv) asymptoten

VA kandidaat $x = 0$ (rechts)

$$\lim_{\substack{x \to 0 \\ >}} \sqrt{x}\ln x = 0 \cdot (-\infty) = \text{ONB}$$

$$= \lim_{\substack{x \to 0 \\ >}} \frac{\ln x}{x^{-1/2}} \overset{H}{=} \lim_{\substack{x \to 0 \\ >}} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}}$$

$$= \lim_{\substack{x \to 0 \\ >}} \left(-2\sqrt{x}\right) = 0 \Rightarrow \text{geen VA}$$

HA $\lim_{x \to +\infty} \sqrt{x}\ln x = +\infty \cdot (+\infty) = +\infty$

$\Rightarrow$ geen HA

SA: $a = \lim_{x \to +\infty} \frac{\sqrt{x}\ln x}{x} = \lim_{x \to +\infty} \frac{\ln x}{\sqrt{x}} \overset{+\infty}{=} \frac{+\infty}{+\infty}$

$$\overset{H}{=} \lim_{x \to +\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x \to +\infty} \frac{2}{\sqrt{x}} = 0 \notin \mathbb{R}(f)$$

$\Rightarrow$ geen SA.

(iv) 1e afgeleide

$$f'(x) = \frac{1}{2\sqrt{x}} \cdot \ln x + \sqrt{x} \cdot \frac{1}{x}$$

$$= \frac{1}{2\sqrt{x}} (\ln x + 2)$$

nulpt$^n$: $\ln x = -2 \Leftrightarrow x = e^{-2}$

polen: $x = 0$

| $x$ | | $0$ | | $e^{-2}$ | |
|---|---|---|---|---|---|
| $\ln x + 2$ | ∄ | − | − | 0 | + |
| $2\sqrt{x}$ | ∄ | + | + | + | + |
| $f'(x)$ | ∄ | − | ∄ | 0 | + |

(v) 2e afgeleide

$$f''(x) = \frac{1}{2} \left( \frac{\ln x + 2}{\sqrt{x}} \right)'$$

$$= \frac{1}{2} \frac{\sqrt{x}\left(\frac{1}{x}\right) - (\ln x + 2)\frac{1}{2\sqrt{x}}}{x}$$

$$= \frac{1}{2} \frac{2 - \ln x - 2}{2x\sqrt{x}}$$

$$= \frac{-\ln x}{4x\sqrt{x}}$$

nulpt$^n$: $x = 1$

polen: $x = 0$

| $x$ | | $0$ | | $1$ | |
|---|---|---|---|---|---|
| $-\ln x$ | ∄ | + | + | 0 | − |
| $4x\sqrt{x}$ | ∄ | 0 | + | + | + |
| $f''(x)$ | ∄ | ∄ | + | 0 | − |

## (vii) Tabel

| $x$ | | $0$ | | $e^{-2}$ | | $1$ | |
|---|---|---|---|---|---|---|---|
| $f'(x)$ | $-$ | $\|$ | $-$ | $0$ | $+$ | $+$ | $+$ |
| $f''(x)$ | $+$ | $\|$ | $+$ | $+$ | $+$ | $0$ | $-$ |
| $f(x)$ | $\searrow$ | $\|$ | $\searrow$ | H.Rkl MIN | $\nearrow$ | Bgpt | $\nearrow$ |

$(e^{-2}, \frac{-2}{e})$ $(1, 0)$

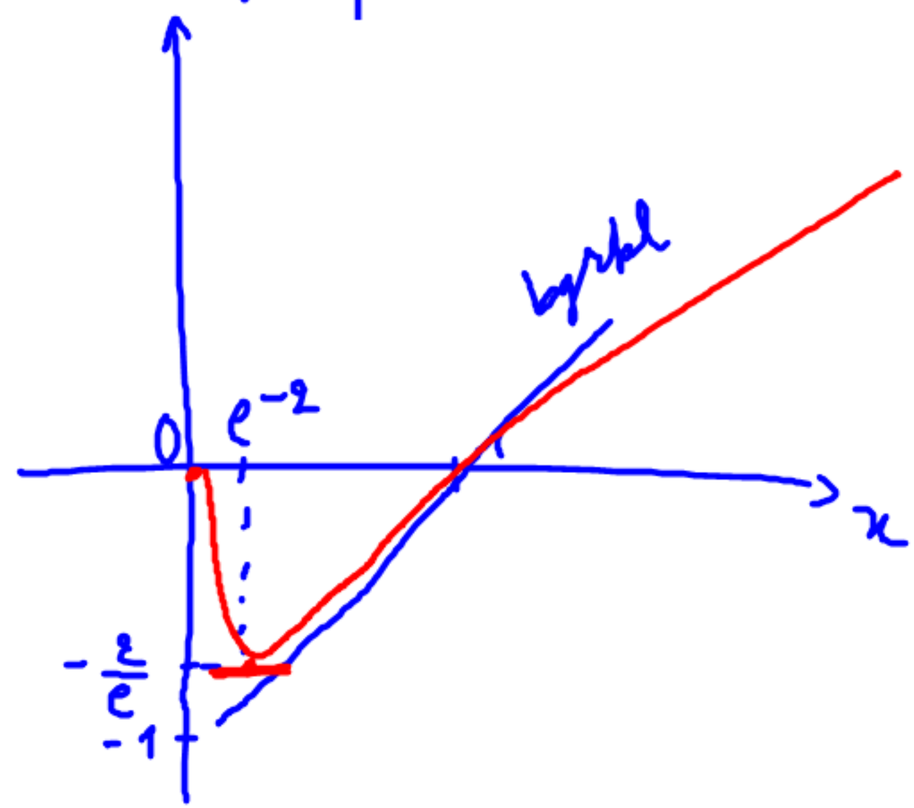
$f(e^{-2}) = e^{-1} \cdot (-2) = -\frac{2}{e}$

rico bgrkl:

$f'(1) = \frac{1}{2}(0+2) = 1$

vgl bgrkl: $y = x - 1$

## (viii) Grafiek




## (ix) Waardenverzameling

$\mathrm{Im} f = \left[-\frac{2}{e}, +\infty\right[$ (zie grafiek)