Oefen. 8(h) p A.15

$$f(x) = \frac{x\sqrt{x+2}}{\sqrt{x+1}}$$

(i) def(f)

VW: $x+2 \geq 0 \Leftrightarrow x \geq -2$

$x+1 > 0 \Leftrightarrow x > -1$

$\text{def}(f) =]-1, +\infty[$

(ii) symmetrie

niet mogelijk want def(f) is niet symmetrisch tov 0.

(iii) Tekenverloop

$\sqrt{} \geq 0$ dus teken van f is het teken van x.

x		-1		0	
$f(x)$	///		$-$	0	$+$

(IV) asymptoten

VA kandidaat $x=-1$ (enkel rechts!)

$$\lim_{\substack{x \to -1 \\ >}} \frac{x\sqrt{x+2}}{\sqrt{x+1}} = \frac{-1.1}{0^+} = -\infty$$

$\Rightarrow$ VA: $x = -1$

HA: $$\lim_{x\to+\infty} \frac{x\sqrt{x+2}}{\sqrt{x+1}} \underset{\downarrow}{=} \lim_{x\to+\infty} \frac{x.\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}} = +\infty$$

hoogste graadstermen!

$\Rightarrow$ geen HA

SA $$a = \lim_{x\to+\infty} \frac{\cancel{x}\sqrt{x+2}}{\cancel{x}\sqrt{x+1}} = 1 \in \mathbb{R}\setminus\{0\}$$

$$b = \lim_{x\to+\infty}\left(\frac{x\sqrt{x+2}}{\sqrt{x+1}} - x\right) = \lim_{x\to+\infty} \frac{x(\sqrt{x+2}-\sqrt{x+1})}{\sqrt{x+1}}$$

$$= \lim_{x\to+\infty} \frac{x(\sqrt{x+2}-\sqrt{x+1})(\sqrt{x+2}+\sqrt{x+1})}{\sqrt{x+1}\,(\sqrt{x+2}+\sqrt{x+1})}$$

$$= \lim_{x \to +\infty} \frac{x\left(\cancel{x}+2-(\cancel{x}+1)\right)}{\sqrt{x+1}\left(\sqrt{x+2}+\sqrt{x+1}\right)}$$

HGTn

$$= \lim_{x \to +\infty} \frac{x}{\sqrt{x}\left(\sqrt{x}+\sqrt{x}\right)}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x}}{2\cancel{x}} \qquad 1$$

$$\Longrightarrow \text{SA}: y = x + \frac{1}{2} \text{ in } +\infty$$

(v) <u>1e afgeleide</u>

$$f'(x) = f(x) \cdot \left(\ln(f(x))\right)'$$

$$= \frac{x\sqrt{x+2}}{\sqrt{x+1}} \left(\ln x + \frac{1}{2}\ln(x+2) - \frac{1}{2}\ln(x+1)\right)'$$

$$= \frac{x\sqrt{x+2}}{\sqrt{x+1}} \left(\frac{1}{x} + \frac{1}{2(x+2)} - \frac{1}{2(x+1)}\right)$$

$$= \frac{\cancel{x}\sqrt{x+2}}{\sqrt{x+1}} \cdot \frac{2(x+2)(x+1)+x(x+1)-x(x+2)}{2\cancel{x}(x+2)(x+1)}$$

$$= \frac{2x^2+5x+4}{2(x+1)\sqrt{x+1}\,\sqrt{x+2}}$$

nulptn: $2x^2+5x+4=0$

$D = 25-4.2.4 = -7 < 0$

$\Rightarrow$ geen nulptn

polen: $x = -1$

x		-1	
$2x^2+5x+4$	$+$		$+$
N		0	$+$
$f'(x)$		|	$+$

leide

$$f''(x) = \frac{2x^2+5x+4}{2\,(x+1)^{3/2}(x+2)^{1/2}} \cdot \left(\ln(2x^2+5x+4) - \frac{3}{2}\ln(x+1) - \frac{1}{2}\ln(x+2)\right)'$$

$$= \frac{2x^2+5x+4}{2\,(x+1)^{3/2}\,(x+2)^{1/2}} \cdot \left(\frac{4x+5}{2x^2+5x+4} - \frac{3}{2(x+1)} - \frac{1}{2(x+2)}\right)$$

$$= \frac{\cancel{2x^2+5x+4}}{2\,(x+1)^{3/2}(x+2)^{1/2}} \cdot \frac{(4x+5)\,2\cdot(x+1)(x+2) - 3(2x^2+5x+4)(x+2) - (2x^2+5x+4)(x+1)}{2\,\cancel{(2x^2+5x+4)}\,(x+1)\,(x+2)}$$

$$= \frac{(8x+10)(x^2+3x+2) - 3(2x^3+9x^2+14x+8) - (2x^3+7x^2+9x+4)}{4\,(x+1)^{5/2}\,(x+2)^{3/2}}$$

$$= \frac{0\cdot x^3 + (34-27-7)x^2 + (46-42-9)x + 20-24-4}{4\,(x+1)^{5/2}\,(x+2)^{3/2}}$$

$$= \frac{-5x-8}{4\sqrt{(x+1)^5}\,\sqrt{(x+2)^3}}$$

nulpt: $x = \frac{-8}{5} < -1$

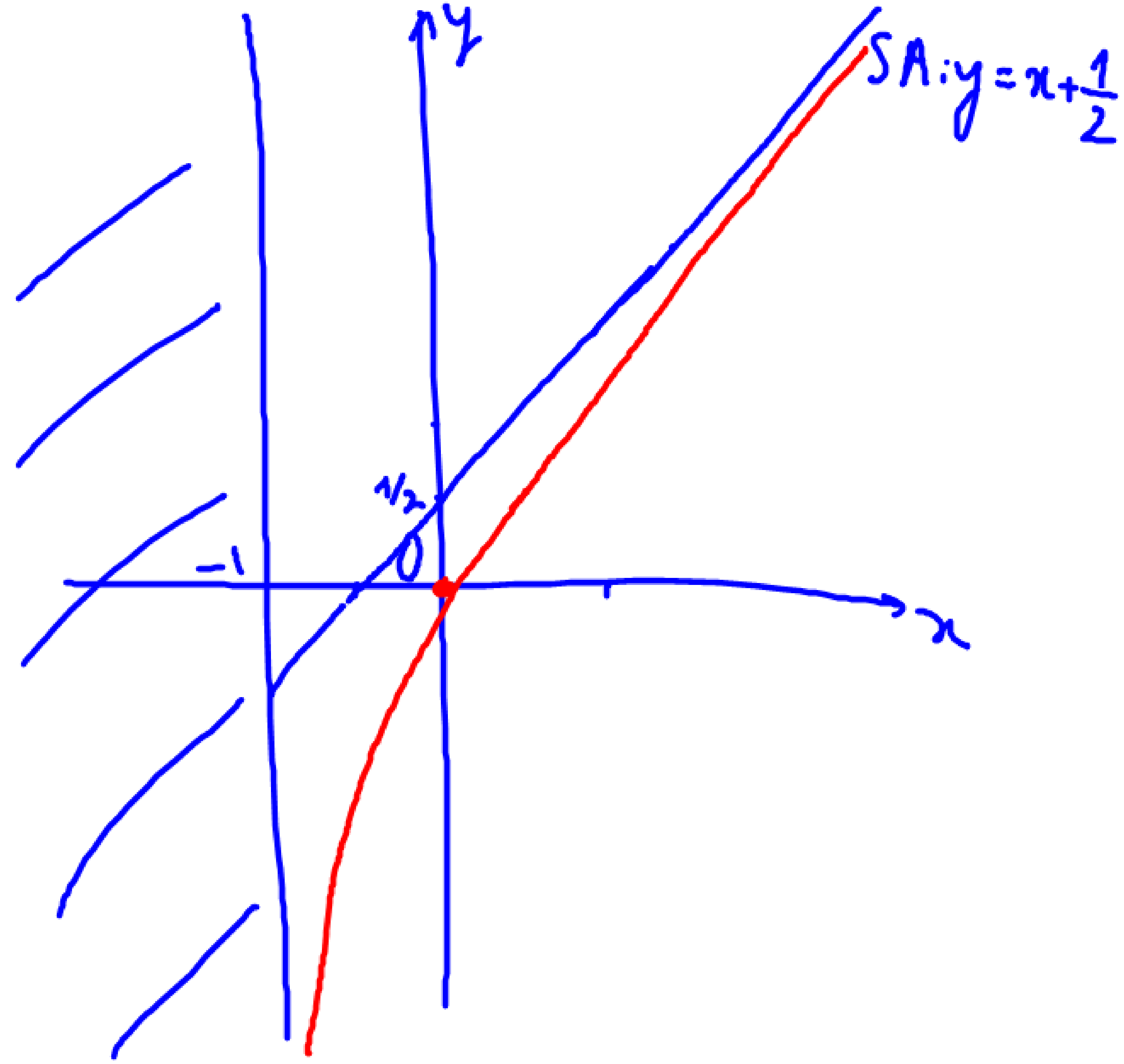
$\Rightarrow$ geen nulpt in def(f)

x		-1	
$f''(x)$		¦	$-$

(vii) Tabel

x		-1		0	
$f'(x)$			$+$	$+$	$+$
$f''(x)$			$-$	$-$	$-$
$f(x)$		$-\infty$	$\nearrow$	0	$\nearrow +\infty$

(viii) grafiek



(ix) waardenverzameling

$\operatorname{Im} f = \mathbb{R}$