Oef 8(f): $f(x) = x e^{-\frac{1}{x}}$

(i) dom(f)

VW: $x \neq 0 \Rightarrow \text{dom}(f) = \mathbb{R} \setminus \{0\}$

(ii) Symmetrie?

$f(-x) = -x e^{\frac{1}{x}} \neq f(x) \Rightarrow$ niet even

$\neq -f(x) \Rightarrow$ niet oneven

(iii) Tekenverloop

$e^{-\frac{1}{x}} > 0$ dus teken van f is teken van x.

x		0	
$xe^{-\frac{1}{x}}$	$-$	|	$+$

(iv) Asymptoten

VA kandidaat: $x = 0$

$$\lim_{x \to 0} x e^{-\frac{1}{x}} = 0 \cdot e^{-\frac{1}{0}}$$

teken belangrijk!

LL: $$\lim_{\substack{x \to 0 \\ <}} x e^{-\frac{1}{x}} = 0 \cdot e^{-\frac{1}{0^-}} = 0 \cdot e^{+\infty} = 0 \cdot +\infty = \text{ONB}$$

$$= \lim_{\substack{x \to 0 \\ <}} \frac{e^{-\frac{1}{x}}}{\frac{1}{x}} \overset{H}{=} \lim_{\substack{x \to 0 \\ <}} \frac{e^{-\frac{1}{x}} \cdot \cancel{\frac{1}{x^2}}}{-\cancel{\frac{1}{x^2}}}$$

$$= \lim_{\substack{x \to 0 \\ <}} \left(-e^{-\frac{1}{x}}\right) = -e^{-\frac{1}{0^-}} = -\infty$$

$\Rightarrow$ VA $x = 0$ langs links

RL: $$\lim_{\substack{x \to 0 \\ >}} x e^{-\frac{1}{x}} = 0 \cdot e^{-\frac{1}{0^+}} = 0 \cdot e^{-\infty}$$

$$= 0 \cdot 0 = 0$$

$\Rightarrow$ geen VA langs rechts

HA $$\lim_{x \to \pm\infty} x e^{-\frac{1}{x}} = \pm\infty \cdot e^0 = \pm\infty$$

$\Rightarrow$ geen HA

- SA: $a = \lim_{x \to \pm\infty} \frac{\cancel{x} e^{-\frac{1}{x}}}{\cancel{x}} = e^{-\frac{1}{\pm\infty}} = e^0 = 1 \in \mathbb{R} \setminus \{0\}$

$b = \lim_{x \to \pm\infty} \left(x e^{-\frac{1}{x}} - x \right) = \lim_{x \to \pm\infty} x \left(e^{-\frac{1}{x}} - 1 \right)$

$\pm\infty \mp \infty = \text{ONB}$ $\qquad$ $\pm\infty \cdot 0 = \text{ONB}$

$= \lim_{x \to \pm\infty} \frac{e^{-\frac{1}{x}} - 1}{1/x} \overset{H}{=} \lim_{x \to \pm\infty} \frac{e^{-\frac{1}{x}} \cdot \cancel{\frac{1}{x^2}}}{-\cancel{1/x^2}} = -1$

$\frac{0}{0}$

$\Rightarrow$ SA: $y = x - 1$ op $\pm\infty$

(V) $\underline{1^e \text{ afgeleide}}$

$f'(x) = e^{-\frac{1}{x}} + x e^{-\frac{1}{x}} \cdot \frac{1}{x^2}$

$= e^{-\frac{1}{x}} \left(1 + \frac{1}{x} \right) = e^{-\frac{1}{x}} \left(\frac{x+1}{x} \right)$

nulpt: $x = -1$

polen: $x = 0$

x		-1		0	
$e^{-\frac{1}{x}}$	$+$		$+$	|	$+$
$\frac{x+1}{x}$	$+$	0	$-$	|	$+$
$f'(x)$	$+$	0	$-$	|	$+$

(VI) $\underline{2^e \text{ afgeleide}}$

$f''(x) = e^{-\frac{1}{x}} \cdot \frac{1}{x^2} \left(1 + \frac{1}{x} \right)$

$+ e^{-\frac{1}{x}} \cdot \left(-\frac{1}{x^2} \right)$

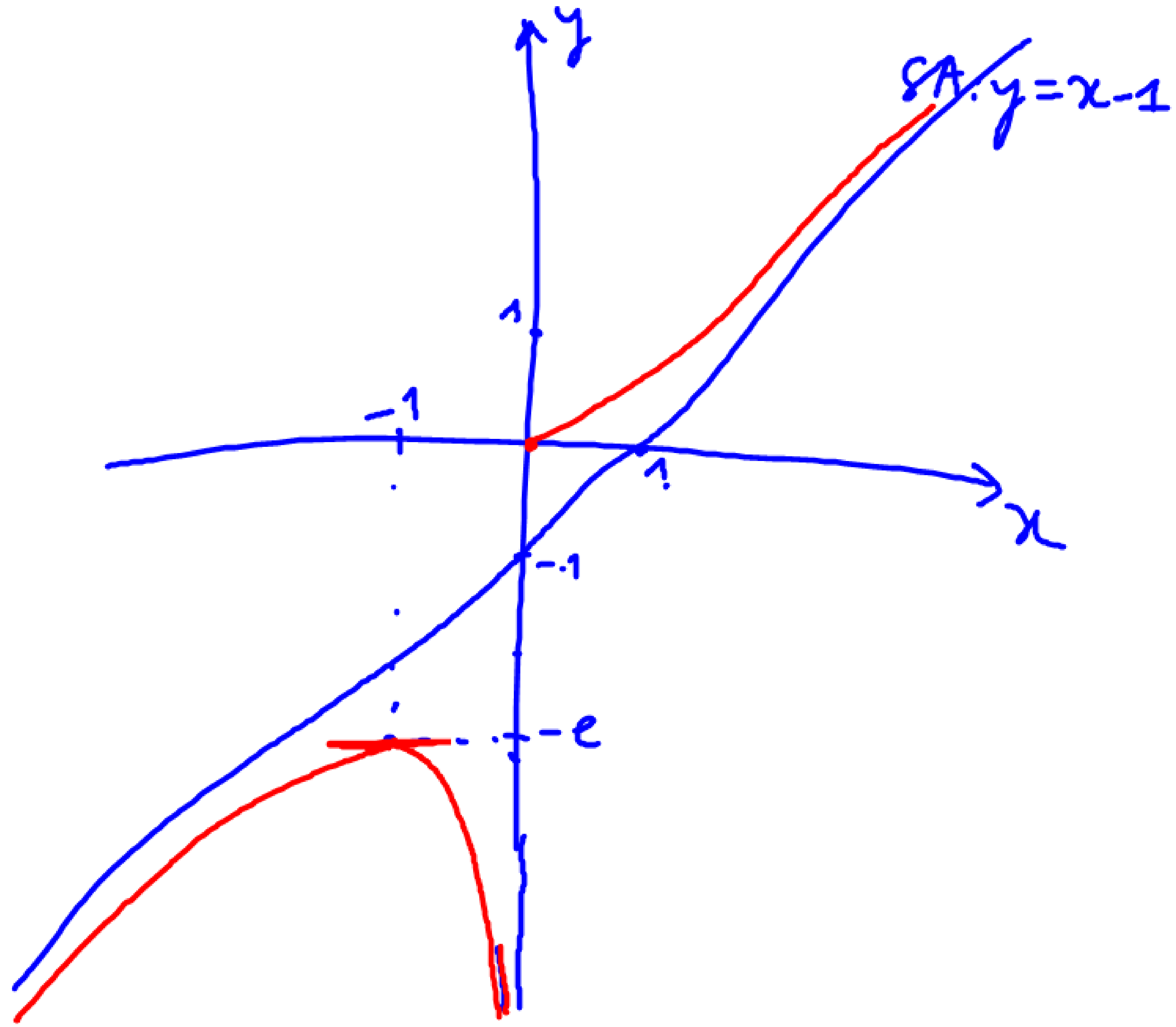
$= \frac{e^{-1/x}}{x^3} + \cancel{\frac{e^{-1/x}}{x^2}} - \cancel{\frac{e^{-1/x}}{x^2}} = \frac{e^{-\frac{1}{x}}}{x^3}$

		0	
$f''(x)$	$-$	|	$+$

(vii) Tabel

x		-1		0	
$f'(x)$	$+$	0	$-$	|	$+$
$f''(x)$	$-$		$-$	|	$+$
$f(x)$	$-\infty \nearrow$	MAX $(-1,-e)$ H.Rkl.	$\searrow -\infty$	|	$0 \nearrow +\infty$

(viii) grafiek



(ix) Waardenverzameling

$\operatorname{Im} f =]-\infty, -e] \cup [0, +\infty[$